TABLE 2 (PART A) Transition system with configurations

(Act$_c$) $\ell \in$

version of this relation, $\dot{\approx}^{\text{ccs}}$

all contexts. To show processes are distinguished it is neccessary to find a live set and a context for which the resulting configurations are not barbed bisimular. These can be found for the the processes P_1 and Q_1, given in the introduction, and therefore they are distinguished by $\approx$. However P_2 and Q_2 are identified though it is far from obvious why. Even worse, processes P_6 and Q_6 (given on page 14) are related, although establishing this fact requi

Proof. (a) follows from Lemma 3.3. (b) is immediate from the definition of strong LF-bisimulation. □

eak LF equiva ence

$C' \stackrel{.}{\approx} D'$. We show that D' must be of the form $\mathbb{C}^{i,j}_{L\setminus\{k\}}[Q']$, up to $\equiv$. From Lemma 3.13 it is easy to see that the liveset in D' must be $L\setminus\{k\}$. To see that the rest of the context must be unchanged, note that C' can silently move to a state in which the only d-actions possible are d_0', d_i', d_0, and d_j. D'q

TABLE 4 Symbolic transition system

$$(\mathsf{Act_s})\ \frac{}{[a.p]_\ell \xrightarrow[\ell]{a} [p]_\ell} \qquad (\mathsf{Tau_s})\ \frac{}{[\tau.p]_\ell \xrightarrow[\ell]{\tau} [p]_\ell}$$

$$(\mathsf{Kill1_s})\ \frac{}{[\mathsf{kill}\, m.p]_\ell \xrightarrow[\ell\wedge\overline{m}]{\tau} [p]_\ell} \qquad (\mathsf{Kill2_s})\ \frac{}{[\mathsf{kill}\, m.p]_\ell \xrightarrow[\ell\wedge m]{kill\, m} [p]_\ell}$$

$$(\mathsf{Live_s})\ \frac{}{[\mathsf{if}\ m}$$

following:

$$\mathrm{after}_{\mu}(\rho) = \begin{cases} \mathsf{ff} & \text{if } \rho \Vdash \mathsf{ff} \\ \mathrm{neg}(\rho) \wedge \overline{k} & \text{if } \mu = \mathit{kill}\, k \\ \mathrm{neg}(\rho) & \text{otherwise} \end{cases}$$

If ρ is unsatisfiable then $\mathrm{after}_{\mu}(\rho)$ is simply ff. Otherwise it corresponds to the *negative information* in ρ; if the action performed is a kill action $\mathit{kill}\, k$, then we must also include the requirement that k be dead, that is, $\overline{k}$.

We now have all the ingredients necessary to give our definition of strong bisimulation equivalence.

DEFINITION 4.5 (STRONG SYMBOLIC BISIMULATION). Let $\mathcal{S}$ be a family of relations on *LProc* indexed by negative formulae ϑ. $\mathcal{S}$ is a *strong symbolic bisimulation* if for every ϑ, $\mathcal{S}_{\vartheta}$ is symmetric and whenever $P \; \mathcal{S}_{\vartheta} \; Q$ and $P \xrightarrow[\pi]{\mu} P'$ then there exist π_i, ρ_i, and Q_i such that for all i,

(a) $\vartheta \wedge \pi \Vdash \bigvee_i \rho_i$, (c) $Q \xrightarrow[\pi_i]{\mu} Q_i$, and
(b) $\rho_i \Vdash \pi_i$, (d) $P' \; \mathcal{S}_{\mathrm{after}_{\mu}(\rho_i)} \; Q_i$

We write $P \simeq^{\mathsf{s}}_{\vartheta} Q$ to indicate that there exists a symbolic bisimulation $\mathcal{S}$ with $P \; \mathcal{S}_{\vartheta} \; Q$. □

For (6a), suppose that $M \vDash \vartheta \wedge \pi$ and therefore (using Equation 4) $P \simeq_M Q$. We show that for some j, $M \vDash \rho_j$. Using the suppositions that $P \xrightarrow[\pi]{\mu} P'$ and $M \vDash \vartheta \wedge \pi$, we can apply the Strong Transition Lemma to conclude that $P \xrightarrow[M]{\mu} P'$ and therefore that there must exist some Q' such that:

$$Q \xrightarrow[M]{\mu} Q' \text{ and } P' \simeq_{\text{iafter}_\mu(M)} Q'$$

By the Strong Transition Lemma there must be some j such that $Q' = Q_j$ and $M \vDash \pi_j$. Because $M \vDash \pi_j$ and $P' \simeq_{\text{iafter}_\mu(M)} Q_j$, we can use (5) to conclude that $M \vDash \rho_j$.

Finally we prove (6b). If $K \vDash \text{after}_\mu(\rho_i)$ then by the definition of "after", there must be some $L \supseteq K$ such that $L \vDash \rho_i$. By (5), $P' \simeq_{\text{iafter}_\mu(L)} Q_i$. Again using the definition of "after", $K \subseteq L$; therefore we may use Lemma 3.5 to conclude, as required, that $P' \simeq_K Q_i$. □

Combining these two Lemmas we obtain the following.

THEOREM 4.8. *$P \simeq_L Q$ if and only if there exists a negative Q-33a4.19t-5.12i-5.12v-4.115*

strong case, the transformation function need not be parameterized by the action μ since the relevant information is already encoded in the temporal formulae.

The formulae we choose as parameters to the relation are simply Boolean

LEMMA 4.13. (a) $P \xRightarrow[\varphi]{\varepsilon} P'$ *if and only if there exist* P_i, π_i *and* h *such that* $1 \le h$, $P_1 = P$, $P_h = P'$, *and the following hold:*

$$\text{for every } 1 \le i < h,\ P_i \xrightarrow[\pi_i]{\tau} P_{i+1},\ \text{and}$$
$$\varphi \dashv\vdash \pi_1 \mathbin{;} \ldots \mathbin{;} \pi_{h-1} \mathbin{;} \mathtt{tt}$$

(b) $P \xRightarrow[\varphi]{\mu} P'$ *if and only if there exist* P_i, π_i, h *and* n *such that* $1 \le h \le n$, $P_1 = P$, $P_{n+1} = P'$, *and the following hold:*

$$\text{for every } 1 \le i \le n,\ i \ne h,\ P_i \xrightarrow[\pi_i]{\tau} P_{i+1},\ \text{and}$$
$$\text{if } \mu = \alpha \quad \text{then } P_h \xrightarrow[\pi_h]{\alpha} P_{h+1} \text{ and } \varphi \dashv\vdash \pi_1 \mathbin{;} \ldots \mathbin{;} \pi_{h-1} \mathbin{;} \pi_h \wedge \pi_{h+1} \wedge \ldots \wedge \pi_n$$
$$\text{if } \mu = \mathit{kill}\,k \text{ then } P_h \xrightarrow[\pi_h]{\mathit{kill}\,k} P_{h+1} \text{ and } \varphi \dashv\vdash \pi_1 \mathbin{;} \ldots \mathbin{;} \pi_{h-1} \mathbin{;} \pi_h \mathbin{;^1} \pi_{h+1} \wedge \ldots \wedge \pi_n$$

Proof. The forward direction (only if) follows by rule induction. The reverse direction follows by induction on n. □

LEMMA 4.14 (WEAK TRANSITION LEMMA).

$$P \overset{\hat{\mu}}{\underset{\mathcal{L}}{\Longmapsto}} P' \text{ if and only if } \exists \psi\colon\ P \xRightarrow[\psi]{\hat{\mu}} P' \text{ and } \mathcal{L} \vDash \psi$$

Proof. In both directions by induction on the definition of weak transitions, using the Strong Transition Lemma and Lemma 4.13. □

The proof of the theorem depends on the following characterisation of LF-bisimulation equivalence (compare Lemma 3.3).

LEMMA 4.15. $\mathcal{S}$ *is a weak LF-bisimulation if and only if for every* L, $\mathcal{S}_L$ *is symmetric and whenever* $P\ \mathcal{S}_L\ Q$*:*

$$\mathcal{L}_{(1)} = L \text{ and } P \overset{\hat{\mu}}{\underset{\mathcal{L}}{\Longmapsto}} P' \text{ imply } \exists Q'\colon\ Q \overset{\hat{\mu}}{\underset{\mathcal{L}}{\Longmapsto}} Q' \text{ and } P'\ \mathcal{S}_{\mathcal{L}_{(\bullet}}\ Q'$$

Proof. Straightforward. □

We now prove the main theorem, treating each direction separately.

PROPOSITION 4.16. *For any Boolean formula formulae* π, *if* $P \approxeq^{\mathsf{s}}_{\pi} Q$ *and* $L \vDash_b \pi$ *then* $P \approxeq_L Q$.

Proof. Let $\mathcal{S}_K$ be defined as follows: $\mathcal{S}_K \overset{\mathit{def}}{=} \{\langle P, Q\rangle \mid \exists \pi\colon\ K \vDash_b \pi \text{ and } P \approxeq^{\mathsf{s}}_{\pi} Q\}$. If $P \approxeq^{\mathsf{s}}_{\pi} Q$ and $L \vDash_b \pi$ then $P\ \mathcal{S}_L\ Q$.

Using the characterisation given above we now show that $\mathcal{S}_K$ is an LF-bisimulation. Suppose that $P\ \mathcal{S}_{\mathcal{L}_{(1}}\ Q$ and therefore we fix a Boolean formula π such that $P \approxeq^{\mathsf{s}}_{\pi} Q$ and $\mathcal{L}_{(1)} \vDash_b \pi$, that is, $\mathcal{L} \vDash \text{initially}(\pi)$. Further suppose that $P \overset{\hat{\mu}}{\underset{\mathcal{L}}{\Longmapsto}} P'$

Basic processes

In this section we turn our attention to the semantics of *basic processes*. In order examine the behaviour of such processes using our operational semantics we need to locate them at a specific site. Moreover it is rather obvious that the choice of this site cannot be ignored. For example, if $p = \mathsf{kill}\,\ell \mid a$, then the meaning of $[p]_\ell$ is different from that of $[p]_k$:

$$[\mathsf{kill}\,\ell \mid a]_\ell \;\dot{\sim}\; [\tau + a.\tau]_\ell \;\dot{\not\approx}\; [\tau \mid a]_\ell \;\dot{\sim}\; [\mathsf{kill}\,\ell \mid a]_k$$

Another example of this is $p = \mathsf{spawn}(\ell, a) \mid b$.

An interesting feature of basic processes is that they determine the semantics of all located processes; any located process P can be translated into a primitive processes p such that $P \approxeq_L [p]_\star$. (For such a translation to hold generally, we believe that the use of the immortal location is essential.) The translation is defined as follows:

$$([p]_\ell)^\bullet = \mathsf{spawn}(\ell, p) \qquad (P\backslash a)^\bullet = P^\bullet\backslash a$$
$$(P \mid Q)^\bullet = P^\bullet \mid Q^\bullet \qquad (P\langle f\rangle)^\bullet = P^\bullet\langle f\rangle$$

THEOREM 5.1. *For any L, $P \approxeq_L [P^\bullet]_\star$*

Proof. By induction on the structure of P. The proof uses the fact that for any L, $[\mathsf{spawn}(\ell, p)]_\star \approxeq_L [\tau.p]_\ell$ and $[\tau.p]_\ell \approxeq_L [p]_\ell$. □

This theorem suggests that it might be appropriate to define a semantic equivalence between basic processes by comparing their behaviour at the immortal site $\star$. However this would ignore important behaviour of processes, namely what they can do when their principal site fails.

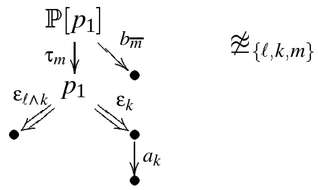
Instead we suggest that the semantics of basic processes should be defined by comparing their behaviour at some arbitrary new locatiion, different from $\star$. The following lemmas show that it the choice of new location does not matter. First a lemma about weak symbolic bisimulation equivalence.

LEMMA 5.2. *Suppose that $\ell \neq \star$. Then $P \approxeq^{\mathsf{s}}_{\vartheta} Q$ implies $P\{^k/_\ell\} \approxeq^{\mathsf{s}}_{\vartheta\{^k/_\ell\}} Q\{^k/_\ell\}$.*

Proof. The proof depends on the following properties of the symbolic operational semantics which are easily established by rule induction.

1. $P \xrightarrow[\varphi]{\mu} Q$ implies $P\{^k/_\ell\}$

consider the context $\mathbb{P}[\cdot] = \mathsf{if}\ m\ \mathsf{then}\ [\cdot]\ \mathsf{else}\ b$. The graphs for $\mathbb{P}[p_1]$ and $\mathbb{P}[q_1]$ are given below:



φ, ψ	§4.3	Temporal formulae
$\overset{\mu}{\underset{\varphi}{\Longrightarrow}}$	§4.3	Weak symbolic transition relation (*LProc* $\times$ *LProc*)
$\mathcal{K}, \mathcal{L}$	§4.3	Live sequence
$\lvert\mathcal{L}\rvert$	§4.3	Length of a live sequence
$\mathcal{L}_{(i)}$	§4.3	i^{th}

- *Local/Global*

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